International Journal of Research in Advent Technology, Vol.6, No.6, June 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

Linear Prime Labeling of Some Direct Snake Graphs

Sunoj B S[1], Mathew Varkey T K[2]

Department of Mathematics, Government Polytechnic College, Attingal, Kerala, India[1]
Department of Mathematics, TKM College of Engineering,Kollam, Kerala, India[2]
spalazhi@yahoo.com[1] *,mathewvarkeytk@gmail.com*[2]

Abstract- Linear prime labeling of a graph is the labeling of the vertices with {0,1,2---,p-1} and the direct edges with twice the value of the terminal vertex plus value of the initial vertex. The greatest common incidence number of a vertex (***gcin***) of in degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of in degree greater than one is one, then the graph admits linear prime labeling. Here we investigated some direct snake graphs for linear prime labeling.

Index Terms- Graph labeling, linear, prime labeling, prime graphs, direct graphs, snake graphs.

1. INTRODUCTION

All graphs in this paper are finite and direct. The direction of the edge is from v_i to v_j iff $f(v_i) < f(v_j)$. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3] and [4] . Some basic concepts are taken from Frank Harary [2]. In this paper we investigated linear prime labeling of some direct snake graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of in degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

Definition 1.2 A Graph is said to be a di graph if each edge of G has a direction**.**

Definition 1.3 In-degree of a vertex in a digraph is the number of edges incident at that vertex.

2. MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection

f : V(G) → {0,1,2,---, p-1} by $f(v_i)$ = i-1, for every i from 1 to p and define a 1-1 mapping f_{lpl}^* : E(G) → set of natural numbers N by $f_{lpl}^*(v_iv_j) = f(v_i) + 2f(v_j)$ for every direct edge v_iv_j. The induced function f_{lpl}^* is said to admit linear prime labeling, if for each vertex of in degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

Definition 2.2 A di graph which admits linear prime labeling is called linear prime di graph.

Theorem 2.1 Direct triangular snake graph T_n (n >2) admits linear prime labeling.

Proof: Let G = T_n and let $v_1,v_2,---,v_{2n-1}$ are the vertices of G.

Here |V(G)| = 2n-1 and |E(G)| = 3n-3.

Define a function f : V → {0,1,2,---,2n-2} by

$f(v_i)$ = i-1 , i = 1,2,---,2n-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{lpl}^* is defined as follows

$f_{lpl}^*(v_{2i-1}\ v_{2i})$ = 6i-4, i = 1,2,---,n-1.
$f_{lpl}^*(v_{2i-1}\ v_{2i+1})$ = 6i-2, i = 1,2,---,n-1.
$f_{lpl}^*(v_{2i}\ v_{2i+1})$ = 6i-1, i = 1,2,---,n-1.

Clearly f_{lpl}^* is an injection.

gcin of (v_{2i+1}) = gcd of $\{f_{lpl}^*(v_{2i-1}\ v_{2i+1}), f_{lpl}^*(v_{2i}\ v_{2i+1})\}$
= gcd of {6i-2, 6i-1}
= 1, i = 1,2,---,n-1.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence T_n, admits linear prime labeling.

Example 2.1 G = T_4.

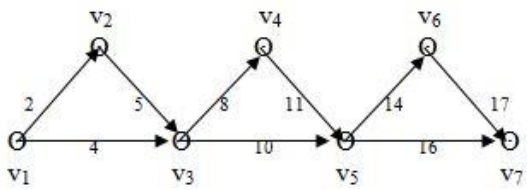

fig – 2.1

Theorem 2.2 Direct alternate triangular snake graph $A(T_n)$ (n >2) admits linear prime labeling, if n is odd and triangle starts from the first vertex.

Proof: Let G = $A(T_n)$ and let $v_1,v_2,---,v_{\frac{3n-1}{2}}$ are the vertices of G.

Here |V(G)| = $\frac{3n-1}{2}$ and |E(G)| = 2n-2.

Define a function f : V → {0,1,2,---, $\frac{3n-3}{2}$ } by

$f(v_i)$ = i-1 , i = 1,2,---, $\frac{3n-1}{2}$.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{lpl}^{*} is defined as follows

$f_{lpl}^{*}(v_{3i-2}\ v_{3i-1})$ = 9i-7, i = 1,2,---,$\frac{n-1}{2}$.

$f_{lpl}^{*}(v_{3i-2}\ v_{3i})$ = 9i-5, i = 1,2,---,$\frac{n-1}{2}$.

$f_{lpl}^{*}(v_{3i-1}\ v_{3i})$ = 9i-4, i = 1,2,---,$\frac{n-1}{2}$.

$f_{lpl}^{*}(v_{3i}\ v_{3i+1})$ = 9i-1, i = 1,2,---,$\frac{n-1}{2}$.

Clearly f_{lpl}^{*} is an injection.

gcin of (v_{3i}) = gcd of {$f_{lpl}^{*}(v_{3i-2}\ v_{3i})$, $f_{lpl}^{*}(v_{3i-1}\ v_{3i})$}
= gcd of {9i-5, 9i-4}
= 1, i = 1,2,---, $\frac{n-1}{2}$.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence A(T_n) ,admits linear prime labeling.

Example 2.2 G = A(T_5).

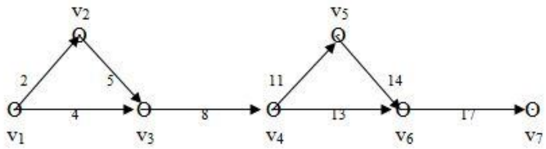

fig – 2.2

Theorem 2.3 Direct alternate triangular snake graph A(T_n) (n >2) admits linear prime labeling, if n is even and triangle starts from the first vertex.

Proof: Let G = A(T_n) and let v_1,v_2,---,$v_{\frac{3n}{2}}$ are the vertices of G.

Here |V(G)| = $\frac{3n}{2}$ and |E(G)| = 2n-1.

Define a function f : V → {0,1,2,---, $\frac{3n-2}{2}$ } by

f(v_i) = i-1 , i = 1,2,---, $\frac{3n}{2}$.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{lpl}^{*} is defined as follows

$f_{lpl}^{*}(v_{3i-2}\ v_{3i-1})$ = 9i-7, i = 1,2,---,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{3i-2}\ v_{3i})$ = 9i-5, i = 1,2,---,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{3i-1}\ v_{3i})$ = 9i-4, i = 1,2,---,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{3i}\ v_{3i+1})$ = 9i-1, i = 1,2,---,$\frac{n-2}{2}$.

Clearly f_{lpl}^{*} is an injection.

gcin of (v_{3i}) = gcd of {$f_{lpl}^{*}(v_{3i-2}\ v_{3i})$, $f_{lpl}^{*}(v_{3i-1}\ v_{3i})$}
= gcd of {9i-5, 9i-4}
= 1, i = 1,2,---, $\frac{n}{2}$.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence A(T_n) ,admits linear prime labeling.

Example 2.3 G = A(T_4).

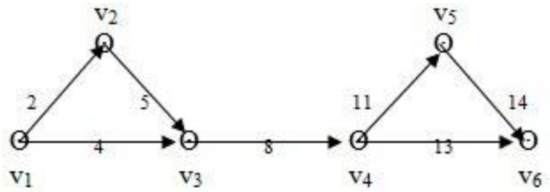

fig – 2.3

Theorem 2.4 Direct alternate triangular snake graph A(T_n) (n >2) admits linear prime labeling, if n is even and triangle starts from the second vertex.

Proof: Let G = A(T_n) and let v_1,v_2,---,$v_{\frac{3n-2}{2}}$ are the vertices of G.

Here |V(G)| = $\frac{3n-2}{2}$ and |E(G)| = 2n-3.

Define a function f : V → {0,1,2,---, $\frac{3n-4}{2}$ } by

f(v_i) = i-1 , i = 1,2,---, $\frac{3n-2}{2}$.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{lpl}^{*} is defined as follows

$f_{lpl}^{*}(v_{3i-2}\ v_{3i-1})$ = 9i-7, i = 1,2,---,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{3i-1}\ v_{3i})$ = 9i-4, i = 1,2,---,$\frac{n-2}{2}$.

$f_{lpl}^{*}(v_{3i-1}\ v_{3i+1})$ = 9i-2, i = 1,2,---,$\frac{n-2}{2}$.

$f_{lpl}^{*}(v_{3i}\ v_{3i+1})$ = 9i-1, i = 1,2,---,$\frac{n-2}{2}$.

Clearly f_{lpl}^{*} is an injection.

gcin of (v_{3i+1}) = gcd of {$f_{lpl}^{*}(v_{3i-1}\ v_{3i+1})$, $f_{lpl}^{*}(v_{3i}\ v_{3i+1})$}
= gcd of {9i-2, 9i-1}
= 1, i = 1,2,---, $\frac{n-2}{2}$.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence A(T_n) ,admits linear prime labeling.

Example 2.4 G = A(T_6).

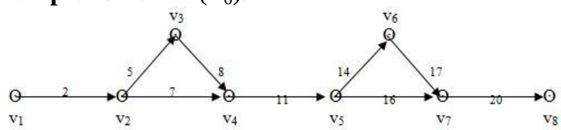

Fig- 2.4

Theorem 2.5 Direct double triangular snake graph DT_n (n >2) admits linear prime labeling.

Proof: Let G = DT_n and let v_1,v_2,---,v_{3n-2} are the vertices of G.

Here |V(G)| = 3n-2 and |E(G)| = 5n-5.

Define a function f : V → {0,1,2,---,3n-3} by

f(v_i) = i-1 , i = 1,2,---,3n-2.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{lpl}^{*} is defined as follows

$f_{lpl}^{*}(v_{3i-2}\ v_{3i-1})$ = 9i-7, i = 1,2,---,n-1.

$f_{lpl}^{*}(v_{3i-2}\ v_{3i})$ = 9i-5, i = 1,2,---,n-1.

$f_{lpl}^{*}(v_{3i-2}\ v_{3i+1})$ = 9i-3, i = 1,2,---,n-1.

$f_{lpl}^{*}(v_{3i-1}\ v_{3i+1})$ = 9i-2, i = 1,2,---,n-1.

$f_{lpl}^{*}(v_{3i}\ v_{3i+1})$ = 9i-1, i = 1,2,---,n-1.

Clearly f_{lpl}^{*} is an injection.

gcin of (v_{3i+1}) $= $ gcd of $\{f^*_{lpl}(v_{3i-1}\ v_{3i+1}),\ f^*_{lpl}(v_{3i}\ v_{3i+1})\}$
$=$ gcd of {9i-2, 9i-1}
$=$ 1, i = 1,2,---,n-1.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence DT_n, admits linear prime labeling.

Example 2.5 G = DT_4.

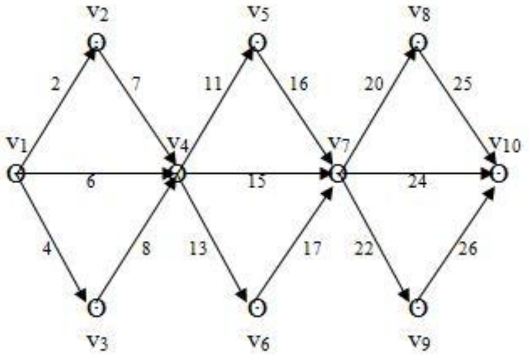

fig – 2.5

Theorem 2.6 Direct alternate double triangular snake graph ADT_n (n >2) admits linear prime labeling, if n is odd and double triangle starts from the first vertex.

Proof: Let G = ADT_n and let $v_1,v_2,---,v_{2n-1}$ are the vertices of G.

Here |V(G)| = 2n-1 and |E(G)| = 3n-3.

Define a function f : V → {0,1,2,---,2n-2} by

$f(v_i)$ = i-1 , i = 1,2,---,2n-1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{lpl} is defined as follows

$f^*_{lpl}(v_{4i-3}\ v_{4i-2})$ = 12i-10, i = 1,2,---,$\frac{n-1}{2}$.

$f^*_{lpl}(v_{4i-3}\ v_{4i-1})$ = 12i-8, i = 1,2,---,$\frac{n-1}{2}$.

$f^*_{lpl}(v_{4i-3}\ v_{4i})$ = 12i-6, i = 1,2,---,$\frac{n-1}{2}$.

$f^*_{lpl}(v_{4i-2}\ v_{4i})$ = 12i-5, i = 1,2,---,$\frac{n-1}{2}$.

$f^*_{lpl}(v_{4i-1}\ v_{4i})$ = 12i-4, i = 1,2,---,$\frac{n-1}{2}$.

$f^*_{lpl}(v_{4i}\ v_{4i+1})$ = 12i-1, i = 1,2,---,$\frac{n-1}{2}$.

Clearly f^*_{lpl} is an injection.

gcin of (v_{4i}) = gcd of $\{f^*_{lpl}(v_{4i-3}\ v_{4i}),\ f^*_{lpl}(v_{4i-2}\ v_{4i})\}$
= gcd of {12i-6, 12i-5}
= 1, i = 1,2,---, $\frac{n-1}{2}$.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence ADT_n, admits linear prime labeling.

Example 2.6 G = ADT_5.

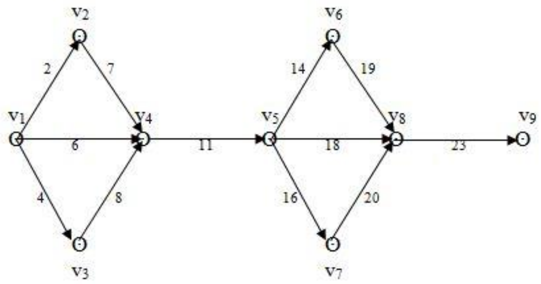

fig – 2.6

Theorem 2.7 Direct alternate double triangular snake graph ADT_n (n >3) admits linear prime labeling, if n is even and double triangle starts from the first vertex.

Proof: Let G = ADT_n and let $v_1,v_2,---,v_{2n}$ are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 3n-1.

Define a function f : V → {0,1,2,---,2n-1} by

$f(v_i)$ = i-1 , i = 1,2,---,2n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{lpl} is defined as follows

$f^*_{lpl}(v_{4i-3}\ v_{4i-2})$ = 12i-10, i = 1,2,---,$\frac{n}{2}$.

$f^*_{lpl}(v_{4i-3}\ v_{4i-1})$ = 12i-8, i = 1,2,---,$\frac{n}{2}$.

$f^*_{lpl}(v_{4i-3}\ v_{4i})$ = 12i-6, i = 1,2,---,$\frac{n}{2}$.

$f^*_{lpl}(v_{4i-2}\ v_{4i})$ = 12i-5, i = 1,2,---,$\frac{n}{2}$.

$f^*_{lpl}(v_{4i-1}\ v_{4i})$ = 12i-4, i = 1,2,---,$\frac{n}{2}$.

$f^*_{lpl}(v_{4i}\ v_{4i+1})$ = 12i-1, i = 1,2,---,$\frac{n-2}{2}$.

Clearly f^*_{lpl} is an injection.

gcin of (v_{4i}) = gcd of $\{f^*_{lpl}(v_{4i-3}\ v_{4i}),\ f^*_{lpl}(v_{4i-2}\ v_{4i})\}$
= gcd of {12i-6, 12i-5}
= 1, i = 1,2,---, $\frac{n}{2}$.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence T_n, admits linear prime labeling.

Example 2.7 G = ADT_4.

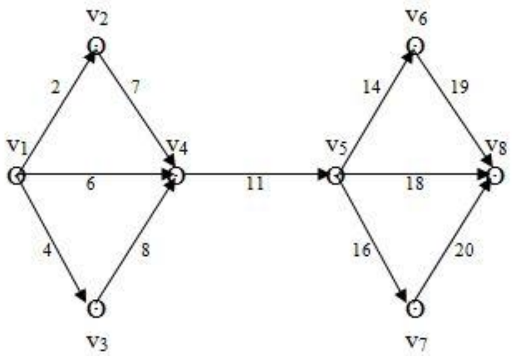

fig – 2.7

Theorem 2.8 Direct alternate double triangular snake graph ADT_n (n >3) admits linear prime labeling, if n is even and double triangle starts from the second vertex.

Proof: Let G = ADT_n and let $v_1, v_2, ---, v_{2n-2}$ are the vertices of G.
Here |V(G)| = 2n-2 and |E(G)| = 3n-5.
Define a function $f : V \rightarrow \{0,1,2,---,2n-3\}$ by

$$f(v_i) = i-1 \text{ , } i = 1,2,---,2n-2.$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{lpl}^* is defined as follows

$f_{lpl}^*(v_{4i-3}\ v_{4i-2})$ = 12i-10, $\quad$ i = 1,2,---,$\frac{n}{2}$.
$f_{lpl}^*(v_{4i-2}\ v_{4i-1})$ = 12i-7, $\quad$ i = 1,2,---,$\frac{n-2}{2}$.
$f_{lpl}^*(v_{4i-2}\ v_{4i})$ = 12i-5, $\quad$ i = 1,2,---,$\frac{n-2}{2}$.
$f_{lpl}^*(v_{4i-2}\ v_{4i+1})$ = 12i-3, $\quad$ i = 1,2,---,$\frac{n-2}{2}$.
$f_{lpl}^*(v_{4i-1}\ v_{4i+1})$ = 12i-2, $\quad$ i = 1,2,---,$\frac{n-2}{2}$.
$f_{lpl}^*(v_{4i}\ v_{4i+1})$ = 12i-1, $\quad$ i = 1,2,---,$\frac{n-2}{2}$.

Clearly f_{lpl}^* is an injection.

gcin of (v_{4i+1}) = gcd of $\{f_{lpl}^*(v_{4i-2}\ v_{4i+1}), f_{lpl}^*(v_{4i-1}\ v_{4i+1})\}$
= gcd of {12i-3, 12i-2}
= 1, $\quad$ i = 1,2,---, $\frac{n-2}{2}$.

So, ***gcin*** of each vertex of in degree greater than one is 1.
Hence ADT_n, admits linear prime labeling.
Example 2.8 G = ADT_6.

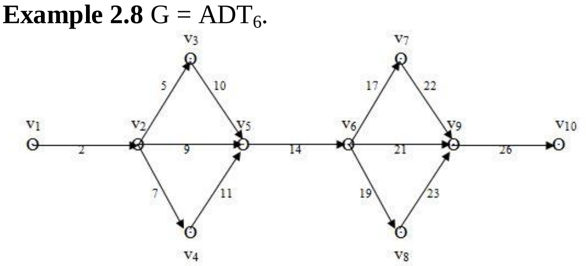

fig – 2.8

3. CONCLUSION

Labeling of direct graphs plays an important role in the study of network related problems. Here we proved that some direct triangular and double triangular snake graphs admit linear prime labeling. Further research can be carried out in labeling of direct triple triangular snake graphs and double quadrilateral snake graphs

REFERENCES

[1] Apostol. Tom M.(1998): Introduction to Analytic Number Theory, Narosa.
[2] F Harary.(1972): Graph Theory, Addison-Wesley,Reading, Mass.
[3] Joseph A Gallian.(2015): A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, #DS6, PP. 1 – 389.
[4] T K Mathew Varkey.(2000): Some Graph Theoretic Operations Associated with Graph Labelings, PhD Thesis,University of Kerala.